

Assignment Quiz 1  
September 22, 1997

Instructor: B.L. Daku  
Time: 15 minutes  
Note: No aids

Name:  
Student Number:

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1. Determine one of the angles of  $x$  (in degrees), where

$$x(n) = \sum_{n=0}^3 \left[ (2)^{\frac{n}{2}} \left( \cos\left(\frac{\pi}{4}n + \frac{\pi}{4}\right) + j \sin\left(\frac{\pi}{4}n + \frac{\pi}{4}\right) \right) \right] \quad (1)$$

$$S = \frac{1 - z^N}{1 - z}$$

$$z = 2^{n/2} \left( \cos\left(\frac{\pi}{4}n + \frac{\pi}{4}\right) + j \sin\left(\frac{\pi}{4}n + \frac{\pi}{4}\right) \right)$$

$$= 2^{n/2} e^{j(\frac{\pi}{4}n + \frac{\pi}{4})}$$

$$S = \frac{1 - z^4}{1 - z} = \frac{1 - 2^{4/2} e^{j(\frac{\pi}{4} \cdot 4 + \frac{\pi}{4})}}{1 - 2^{0/2} e^{j(\frac{\pi}{4} \cdot 0 + \frac{\pi}{4})}}$$

$$= \frac{1 - 4 \cdot e^{j5\pi/4}}{1 - \sqrt{2} e^{j\pi/4}}$$

$$= \frac{1 - 4 \cos(5\pi/4) - j4 \sin(5\pi/4)}{1 - \sqrt{2} \cos(\pi/4) - j\sqrt{2} \sin(\pi/4)}$$

$$= \frac{1 + 2\sqrt{2} + j2\sqrt{2}}{1 - j\sqrt{2}}$$

$$= \frac{3.8284 + j2.8284}{1 - j1.4142}$$

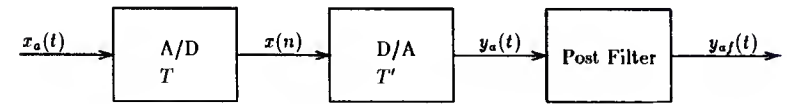
$$= \frac{4.7599 \angle 36.45^\circ}{.3333 \angle -47.7^\circ}$$

$$= 8.2444 \angle -18.279^\circ$$

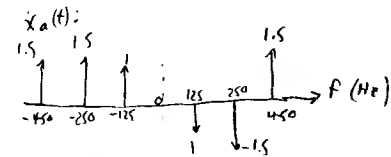
1. Consider the simple signal processing system shown in the following figure. The sampling periods of the A/D and D/A converters are  $T = 5$  ms and  $T' = 1$  ms, respectively. Determine  $x(n)$ ,  $y_a(t)$  and  $y_f(t)$  of the system, if the input is

$$x_a(t) = 3 \cos(900\pi t) + 2 \sin(250\pi t) + 3 \sin(500\pi t) \quad (1)$$

The postfilter removes any frequency component above  $\frac{F_s}{2}$ , where  $F_s = \frac{1}{T}$ .



A/D:  
 $T = 5$  ms  
 $\therefore F_s = 200$  samples/s



$$f_s = \frac{1}{T} = 200$$

$$t = \frac{n}{F_s}$$

$$X(n) = 3 \cos\left(2\pi \left(\frac{900}{200}\right) n\right) + 2 \sin\left(2\pi \left(\frac{250}{200}\right) n\right) + 3 \sin\left(2\pi \left(\frac{500}{200}\right) n\right)$$

$$= 3 \cos(2\pi n \frac{9}{4}) + 2 \sin(2\pi n \frac{5}{8}) + 3 \sin(2\pi n \frac{5}{4})$$

$$= 3 \cos(2\pi n \frac{11}{4}) + 2 \sin(2\pi n \frac{5}{8}) + 3 \sin(2\pi n \frac{1}{4})$$

$$X(n) = 3 \cos(2\pi n \frac{11}{4}) - 2 \sin(2\pi n \frac{3}{8}) + 3 \sin(2\pi n \frac{1}{4})$$

$n = t \cdot F_s$

$$\therefore y_a(t) = 3 \cos(2\pi t \frac{900}{4}) - 2 \sin(2\pi t \frac{250}{8}) + 3 \sin(2\pi t \frac{500}{4})$$

$$y_a(t) = 3 \cos(100\pi t) - 2 \sin(150\pi t) + 3 \sin(100\pi t)$$

\* filter removes freq above  $\frac{F_s}{2} = \frac{1}{T \cdot 2} = 100$  Hz; \* all freq of  $y_a(t)$  are below

$$y_f(t) = 3 \cos(100\pi t) - 2 \sin(150\pi t) + 3 \sin(100\pi t)$$

$$= y_a(t)$$